## Appendix: Some Mathematics

*How should a group of people make decisions?*

Suppose we have a group of *n* people who have all agreed to pool resources and work towards a common cause. In the process of doing so, they will face a series of decisions on which, even after thorough discussion, their opinions will vary. The correct decision is usually clear in hindsight, if the common cause is well-articulated, but not in advance. Each person individually will be right some percentage of the time, a measure of *competency* varying from person to person. How should they best aggregate their opinions so as to maximize the percentage of decisions in which the group as a whole acts correctly?

The two historical solutions to this problem are *leaders* and *voting.* In the leader approach, the group examines past decisions and determines which person was most consistently correct. That person then makes the decision. For best results, the leader's record should be frequently reassessed, and if found wanting the incompetent CEO is fired, or the tree of liberty is refreshed with the blood of tyrants, or something. In the voting approach, all *n* people, or some subset judged to have a particularly large average competency, record their opinions, and the course of action favored by the majority wins. This is well-known to be the worst form of government, excluding all those others that have been tried.

But we have computers now! The question of how to aggregate different people's opinions, given varying competencies, reduces to the largely solved problem of weighting the inputs given to an artificial intelligence. When viewed as decision-making algorithms, the concepts of leaders and majority votes are revealed as naïve and simplistic; arbitrary points in a vast solution space. To illustrate, let's consider the simplest interesting problem in group decision-making: three people **A**, **B**, and **C**, among whom **C** is the most competent, and whose decisions are not otherwise correlated with each other's. When **A** and **B** agree with each other but disagree with **C**, whose opinion should be followed? This is not a subjective question! We can formalize it as follows:

We define the competency of person **X**, or $c_x$, as the proportion of the time that **X**'s decisions are correct. If **X** is always right, $c_x = 1$, while if **X** is right seven times out of ten, $c_x =$

.7.  We will assume that all competencies are greater than ½, since the advice of someone who is wrong the majority of the time will be ignored or inverted.  In this case, we have competencies $c_A$, $c_B$, and $c_C$.  When **A** and **B** agree, by the law of independent probability they will be wrong $(1- c_A)*(1- c_B)$ of the time, while **C** will be wrong $(1- c_C)$ of the time.  Thus **C** should be obeyed if and only if  $(1- c_C)<(1- c_A)*(1- c_B)$.  For example, if **A** and **B** are each right .6 of the time, then when they agree they will be right [1 – (1-.6)*(1-.6)]=.84 of the time.  Thus they should defer to **C** if and only if **C** is right more than 84% of the time.

Determining the competency of a voter is also reducible to a simple well-known result in probability theory.  We look only at the previous decisions that the voter has voted on and whose wisdom or lack thereof is evident in hindsight.  Then by Laplace's Rule of Succession, if a voter has made **g** good decisions and **b** bad ones, the likelihood that the voter's next decision will be good is given by (**g+1)/(g+b+2)**.  Note that this reflects the intuition that someone with a long record is more reliable than than someone with a similar success rate but shorter record.  Someone who has been correct in 7 out of 8 cases has a competency of (7+1)/(8+2) = .8, while someone who has been correct in 77 out of 88 cases has a competency of .8666666...

So far we have merely provided a criterion for distinguishing between leader-appropriate and voter-appropriate situations.  But now add a fourth decision-maker, **D,** with a greater competency than **C**.  If $(1-c_D)<(1- c_A)*(1- c_B)*(1- c_C)$, **D** should be the leader, but otherwise a leader approach is not optimal and the various 2 vs. 2 disagreements must be considered.  Given our assumptions, we can optimally choose between the action favored by **B** and **C** over the action favored by **A** and **D** when and only when $(1- c_B)*(1- c_C)<(1- c_A)*(1- c_D)$.

In general, simple probability theory has given us a method of making group decisions that, given our assumptions, is provably optimal, and does not always correspond to traditional methods.  We should choose the opinions of people **Y1, Y2, Y3...** over those of people **N1, N2, N3...** if and only if

$$_{i}(1-c_{Yi}) < \quad _{i}(1-c_{Ni})$$

Notably, this condition is equivalent to

$$ {}_i\ -\log(1-c_{Yi}) > {}_i\ -\log(1-c_{Ni}) $$

This means that the optimal method will always be expressible as a *weighted vote,* in which each voter's weight can be individually determined by the voter's competency. In the three-voter case, the formula might give us weights of (.6,.6,.8), which would equate to a simple voting rule since .6+.6 > .8, or it could give us (.6,.6,1.3), effectively making **C** the leader. Following the naming convention that gives us democracy, aristocracy, and so forth we can call this method *histocracy*, rule by a system of weights.

This optimality proof does not hold if we allow votes to be correlated, as they generally are. If **A** and **B** always vote together and are correct 60% of the time, they should not properly be able to override a **C** with a competence of .8. Tools (e.g. a perceptron algorithm) exist to solve this problem. However, abandoning the principle of determining a voter's weight based solely on that individual's past results risks incentivizing strategic voting. A subgroup with goals differing from the group's charter could inflate its voting share by intentionally splitting its votes on unimportant issues. Whether a simple weight system or a more sophisticated algorithm is appropriate thus depends on the group.